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Comments on the complex linear Goldstino superfield

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Abstract

It is shown that the complex linear Goldstino superfield recently proposed in arXiv:1507.01885 is obtained from the one originally constructed in arXiv:1102.3042 by a field redefinition.

Four years ago, we constructed the Goldstino model [1] described by a *modified* complex linear superfield Σ ,

$$-\frac{1}{4}\bar{D}^2\Sigma = f, \quad f = \text{const}. \quad (1)$$

Here f is a parameter of mass dimension 2 which, without loss of generality, can be chosen to be real. To describe the Goldstino dynamics, Σ was subject to the following nonlinear constraints:

$$\Sigma^2 = 0, \quad (2)$$

$$-\frac{1}{4}\Sigma\bar{D}^2D_\alpha\Sigma = fD_\alpha\Sigma. \quad (3)$$

The constraint (2) tells us that Σ is nilpotent. The form of the Goldstino action coincides with the free action for the complex linear superfield,

$$S[\Sigma, \bar{\Sigma}] = - \int d^4x d^2\theta d^2\bar{\theta} \Sigma \bar{\Sigma}. \quad (4)$$

It was also shown in [1] that all known Goldstino superfields can be obtained as composite constructed from spinor covariant derivatives of Σ and its conjugate $\bar{\Sigma}$.¹ Such constructions make use of the spinor superfields Ξ_α and its conjugate $\bar{\Xi}_{\dot{\alpha}}$ defined by

$$\Xi_\alpha = \frac{1}{\sqrt{2}}D_\alpha\bar{\Sigma}, \quad \bar{\Xi}_{\dot{\alpha}} = \frac{1}{\sqrt{2}}\bar{D}_{\dot{\alpha}}\Sigma. \quad (5)$$

Making use of the constraints (1), (2) and (3), we can readily uncover those constraints which are obeyed by the above spinor superfields. They have the form (with $2\kappa^2 = f^{-2}$)

$$\bar{D}_{\dot{\alpha}}\bar{\Xi}_{\dot{\beta}} = \kappa^{-1}\varepsilon_{\dot{\alpha}\dot{\beta}}, \quad (6a)$$

$$D_\alpha\bar{\Xi}_{\dot{\alpha}} = 2i\kappa\bar{\Xi}^{\dot{\beta}}\partial_{\alpha\dot{\beta}}\bar{\Xi}_{\dot{\alpha}} \quad (6b)$$

and are exactly the constraints given in [5], so we recognise Ξ_α as the Samuel-Wess superfield. In particular, for the Goldstino superfield Σ the solution to the constraints (1), (2) and (3) in terms of $\bar{\Xi}_{\dot{\alpha}}$ is very simple:

$$2f\Sigma = \bar{\Xi}^2, \quad (7)$$

and the action (4) takes the form

$$S[\Sigma, \bar{\Sigma}] = -\frac{1}{4f^2} \int d^4x d^2\theta d^2\bar{\theta} \bar{\Xi}^2\bar{\Xi}^2. \quad (8)$$

¹This property and the universality [2, 3, 4] of the Goldstino [2] implies that any model for supersymmetry breaking can be described in terms of Σ and its conjugate.

One option for a composite Goldstino superfield that was not discussed in [1] was recently explored in [6]. It is defined in terms of the Samuel-Wess superfield as

$$\Gamma = -\frac{\kappa^2}{\sqrt{8}}\bar{D}_{\dot{\alpha}}(\bar{\Xi}^{\dot{\alpha}}\Xi^2) . \quad (9)$$

Making use of (6) gives

$$\Gamma = \frac{\kappa}{\sqrt{2}}\Xi^2\left\{1 - \kappa^2 i\bar{\Xi}_{\dot{\alpha}}\partial^{\dot{\alpha}\alpha}\Xi_{\alpha}\right\} . \quad (10)$$

By construction, Γ is an *ordinary* complex linear superfield,

$$\bar{D}^2\Gamma = 0 , \quad (11)$$

and it follows from (8) and (10) that it has an identical action to Σ ,

$$S[\Sigma, \bar{\Sigma}] = S[\Gamma, \bar{\Gamma}] , \quad (12)$$

where $S[\Gamma, \bar{\Gamma}]$ is obtained from (4) by replacing $\Sigma \rightarrow \Gamma$. As a consequence of (10), Γ is nilpotent,

$$\Gamma^2 = 0 . \quad (13)$$

Equations (6a) and (6b) imply that Γ satisfies a nonlinear constraint that was not discussed in [6]

$$-\frac{1}{4}\Gamma\bar{D}^2\bar{\Gamma} = f\Gamma . \quad (14)$$

This is structurally identical to the second constraint of Rocek [7], which is natural since Γ is on-shell equivalent to Rocek's constrained chiral superfield [6]. As this constraint mixes Γ with its conjugate, it leads to complicated solutions for its components. This is in contradistinction to the constraint (3) that only depends on Σ . The solutions to the constraint (14) and all of the calculations in this paragraph can be found in the ancilliary Mathematica notebook attached to [8].

It is interesting to note that in the *modified* complex linear superfield of [1] the constraints mean that the Goldstino arises from the normally physical fermion $G_{\alpha} \propto D_{\alpha}\bar{\Sigma}$. In the complex linear superfield Γ , the constraints mean that the Goldstino arises from a normally auxiliary spinor $G_{\alpha} \propto D_{\alpha}\Gamma$. This was discussed in [6] and is now easily seen from (10) when rewritten using (5) as

$$\Gamma = \bar{\Sigma} - \frac{\kappa}{\sqrt{8}}(\bar{D}_{\dot{\alpha}}\Sigma)(\bar{D}^{\dot{\alpha}}\bar{\Sigma}) . \quad (15)$$

In conclusion, we would like to point out that the Σ -realisation [1] and the Γ -realisation [6] for complex linear Goldstino superfield are to some extent complementary. The former is characterised by the holomorphic-like constraint (3) that is very useful for applications. The latter has been deduced from models allowing spontaneous supersymmetry breaking. It is worth mentioning that the constraint (1) is the only way to describe $\mathcal{N} = 1$ anti-de Sitter supergravity using a non-minimal scalar multiplet [9].

References

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